Fourth Semester B.E. Degree Examination, December 2011

Signals and Systems

Time: 3 hrs.

Max. Marks:100

Note: Answer any FIVE full questions, selecting at least TWO questions from each part.

 $\frac{PART-A}{a.\quad A \ continuous-time \ signal \ x(t) \ is \ shown \ in Fig.Q1(a). \ Sketch \ and \ label \ each \ of the \ following:$

i) x(t)u(1-t)

ii) x(t)[u(t) - u(t-1)]

iii) $x(t) \delta \left(t - \frac{3}{2}\right)$

iv) x(t) [u(t+1)-4(t)]

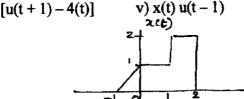


Fig.Q1(a)

b. Consider the following sinusoidal signal. Determine whether each x(n) is periodic and if it is find its fundamental period.

i) $x(n) = 10 \sin(2n)$

- ii) $x(n) = 15 \cos(0.2\pi n)$ iii) $x(n) = 5 \sin[6\pi n/35]$

(06 Marks)

(10 Marks)

c. If $x(n) = \{1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7\}$, find: i) y(n) = x(2n-3), ii) y(n) = x(-2n+1)

(04 Marks)

a. Find the convolution of x(t) with h(t), where 2

x(t) = A[u(t) + u(t - T)] and h(t) = A[u(t) - u(t - 2T)]

(10 Marks)

- b. A discrete system has impulse response $h(n) = a^n u(n+3)$. Is this system BIBO stable, (03 Marks) causal and memory less?
- c. The impulse response of the system is given by $h(t) = e^{-2|t|}$, find the step response of the (07 Marks) system.
- Determine the condition of the impulse response of the system is :

i) memory less

- ii) causal
- iii) stable
- iv) invertible.

(10 Marks)

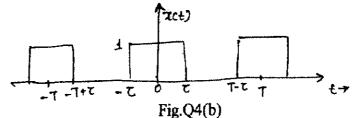
- b. Solve the differential equation : $\frac{d^2y(t)}{dt^2} + 3\frac{dy(t)}{dt} + 2y(t) = 2x(t)$, with initial conditions (10 Marks) y(0) = 0, y'(0) = 1 for the input $x(t) = \cos t u(t)$.
- Determine the Fourier series representation of the following signals:

i)
$$x(t) = 3\cos\left[\frac{\pi}{2}t + \frac{\pi}{4}\right]$$

ii) $x(t) = 2\sin(2\pi t - 3) + \sin 6\pi t$

(10 Marks)

Determine the Fourier series representation for the square wave shown in Fig.Q4(b).



(10 Marks)

PART - B

- a. Use the differentiation in time and differentiation in frequency properties to determine the FT of Gaussian pulse defined by $g(t) = \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}}$. (10 Marks)
 - b. Find the FT of $x(t) = \frac{1}{1+it}$. (05 Marks)
 - c. Find the inverse FT of $x(j\omega) = \frac{(1-j\omega)}{6+j\omega+\omega^2}$. (05 Marks)
- (08 Marks) State and prove Rayleigh's energy theorem. 6
 - Find the frequency response and impulse response of the system with input x(t) and output y(t) is given by:
 - i) $x(t) = e^{-2t} u(t)$ and $y(t) = e^{-3t} u(t)$ ii) $x(t) = e^{-2t} u(t)$ and $y(t) = 2t e^{-2t} u(t)$ c. Determine the difference equation description for the system with impulse response.
 - $h(n) = 3\delta(n) + 2\left(\frac{1}{2}\right)^n u(n) + \left(-\frac{1}{2}\right)^n u(n)$ (04 Marks)
- a. Determine the ZT of the following sequence:
 - $ii) x(n) = n^2 u(n).$ i) $x(n) = \alpha^{|n|}$ for $|\alpha| < 1$ b. Find the inverse ZT of: (10 Marks)
 - - i) $x(z) = \frac{16z^2 4z + 1}{8z^2 + 2z 1}$ for $|z| > \frac{1}{2}$ ii) $x(z) = e^{z^2}$ for all $z |z| \neq \infty$. (10 Marks)
- A system has the transfer function 8

H(z) =
$$\frac{2}{1 - 0.9e^{j\frac{\pi}{4}}z^{-1}} + \frac{2}{1 - 0.9e^{j\frac{\pi}{4}}z^{-1}} + \frac{3}{1 + 2z^{-1}}$$
.

Find the impulse response assuming the system is (i) stable and (ii) causal. (10 Marks)

b. A system is described by the difference equation:

$$y(n) - y(n-1) + \frac{1}{4}y(n-2) = x(n) + \frac{1}{4}x(n-1) - \frac{1}{8}x(n-2)$$
.

(05 Marks) Find the transfer function of the system.

(05 Marks) c. State and prove final value theorem in ZT.